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LEVEL III

OPTIMIZATION PROBLEMS: DUALITY AND MULTIPLIER METHODS

Grant AFOSR-77-3204

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The main accomplishment in 1979, the third year under grant AF-AFOSR-77-3204, was the completion of the manuscript of the book (1). The final push involved adding over 200 pages to the 500 plus that had been put together earlier. With this out of the way, attention turned to research on several topics described below. Results of this research are now being written up for publication in technical journals. This year 1979 also saw the publication of a number of articles that had been written in the preceeding grant period (2), (3), (4), (5), (6).

Monotropic programming. This is the term I have started using to describe the area of optimization theory that includes linear programming, network programming and separable convex programming, and which is characterized by solution methods that have a distinctly combinatorial nature: descent is in special directions induced by a matroidal substructure. This area has until now not been investigated or even recognized as a unified whole, and herein lies the novelty and significance of the monograph (1). For a better idea of the contribution, the preface to (1), the table of contents and the section of comments at the end of each chapter will provide an overview. Many new computational methods and conceptual innovations are provided. The book includes the first comprehensive treatment of nonlinear network flow problems and separable convex programming, and incidentally it was these topics that demanded most of the writing effort in 1979.

Now that the book is finished, I have turned for the time being to research primarily in other directions. The subject is by no means exhausted, however, and I intend to return to it. A specific question which intrigues me, and which I continue to work at off and on, is whether the general "out-of-kilter" algorithm that I devised for separable convex programming problems with piecewise linear cost terms can be extended to the piecewise quadratic case. If so, one will have obtained a highly efficient method that can be adapted to the approximate solution of still more general problems. The trouble with all the truly "quadratic" type methods obtained so far for nonlinear networks and other problems in monotropic programming is that they do violence to the special structure of such problems, or at least fail to take appropriate advantage of it. They just don't seem apt.

Subgradient optimization. A lot was said in last year's interim report about the importance of this subject and the motivation for it. My own interest in the area stems naturally from my work over the years in convex analysis. What that taught me was the feasibility and great usefulness of developing methods of analysis, inspired by applications to optimization problems, that could mimic classical calculus in treating functions that were not necessarily differentiable. I learned that such functions are much more common in optimization than might at first be supposed, especially in secondary constructs, and that the technical ability to handle them can bring about a remarkable simplification of outlook that stimulates fresh thinking.

The task nowadays is to generalize this approach to nonconvex situations. The chief breakthroughwas provided in 1973 by Frank Clarke in his thesis written under me (and supported by AFOSR). Clarke's ideas have had a strong impact on the work of many people in optimization. As for myself, I labored long and hard in 1978 to put these ideas in a broader and more flexible framework and wrote a number of articles that have this year appeared in print [2], [4], [6].

At present, I am exploring the question of how to characterize the subgradients and generalized directional derivatives of the function $p(v) = \inf(P_v)$ giving the optimal value in a nonlinear programming problem that depends on a parameter vector $v = (v_1, \ldots, v_d)$. I have some theorems expressing these things in terms of optimal solutions and Lagrange multipliers in (P_v). Applications include decomposition methods where (P_v) appears as a subproblem and $p(v)$ must be optimized by the right choice of v. This work will wind up in technical articles submitted for publication in 1980.

Open horizon control. Another topic I devoted some attention to in 1979 is that of control problems over time intervals of the form $[t_0, t_1)$ or $(t_0, t_1]$. Dropping one of the endpoints of the interval may not at first seem like a significant step, but it throws the mathematics into a much more difficult mode. The motivation is not simply technical, however; there are problems in engineering and economics that lead naturally to this case. The meaning of the absence of one of the endpoints of the time interval is that in determining an optimal trajectory, no a priori condition at all is to be imposed on the behavior of the trajectory as time approaches that endpoint (possibly infinite). The trajectory need not even be bounded, although it may well be that boundedness turns out to be dictated by optimality. This point of view is important in studying the behavior of systems near an equilibrium. It is undesirable in such a context to have to pick a (finite) terminal time and say what the state should be at that time. Rather, one wants to discover what behavior patterns will be dictated by optimality alone when the future is always kept open, at least by ϵ.

The work I did was concerned with an existence theorem for optimal trajectories in such situations. Classical results are inapplicable. I got such a theorem and plan to write it up for publication in the coming year. More than just existence is at stake in such work, of course, since the true task is in fact to identify the kinds of conditions under which the problem really is sensible. With this achieved, I hope it will be possible to make progress on the nature of the optimality conditions for such problems. The question of what "transversality" condition to use at the missing endpoint has been a great mystery to many people, but I have an inkling now of how an answer can be attempted.

(1) Optimization in Networks and Monotropic Systems, 750 page manuscript (being published by Princeton University Press).

(2) "Clarke's tangent cones and the boundaries of closed sets in R", Nonlin. Analysis, TR. Meth. Appl. 3 (1979), 145-154.

(3) "Convex processes and Hamiltonian dynamical systems", in Convex Analysis and Math. Economics, Springer-Verleg lecture notes no. 168 (1979), 122-126.

(4) "Directionally Lipschitzian functions and subdifferential calculus", Proc. London Math. Soc. 33 (1979), 331-335.

(5) "The generic nature of optimality conditions in nonlinear programming", Math. of O.R. 4 (1979), 425-430.

(6) "Generalized directional derivatives and subgradients of nonconvex functions", Canadian J. Math.

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This report describes the research under this grant in 1979. A monograph on network optimization and monotropic programming was completed. New results were obtained on subradient methods and open horizon control.